Building Conceptual Understanding of the Fluency Expectations

Grades K-6

Math facts fluency has always been an expectation for students. Fluency, or quick and accurate retrieval of basic computational math facts, is a necessary skill. The strategies to build fluency are, however, debated. Historically many teachers have utilized the “drill and practice” methods without building conceptual understanding. On the other hand, some build a strong conceptual understanding without the practice. The Indiana State Standards require teachers to make a shift in thinking that addresses the need for fluency through building a deep conceptual understanding. It is the marriage between the two that will be essential to build strong computational skills in students. In the following pages, you will find the fluency expectations for the Indiana State Standards along with examples and ideas on how to increase math fluency through conceptual understanding.

Conceptual understanding requires students to develop a comprehension of math concepts and operations, integrate mathematical ideas, and apply them in different situations. Often times, students can grasp a concept through pictorial representations, manipulatives, hands-on models, and diagrams. To further their understanding, they integrate this knowledge to new concepts by comparing and contrasting methods and relationships. Students can explain their methods and reconstruct their methods to new problems. They can also generate examples and non-examples of the concept. It is no longer enough to simply memorize basic computational facts, but rather that students can explain their understanding and apply it to alternate situations.

According to the Indiana State Standards, the standards provided within this document are designated as fluency expectations for each grade level. By the end of the grade, students should be computing quickly and proficiently. Please note that there are no guidelines for “timing” a student. The standards dictate that a student can answer the question with automaticity rather than writing answers to problems in a timed situation. Helping children develop mastery with facts requires a continuum of learning. First, they must develop an understanding of number
relationships then develop efficient strategies to retrieve answers to facts. Ultimately, drill and practice is the goal to assess fluency, however; this drill and practice cannot occur PRIOR to building conceptual understanding (Van de Walle, 2005).

<table>
<thead>
<tr>
<th>Grade Level</th>
<th>Fluency Expectations</th>
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<tbody>
<tr>
<td>K</td>
<td>K.CA.1 Add and subtract within 10</td>
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<tr>
<td>1</td>
<td>1.CA.1 Add and subtract within 20</td>
</tr>
<tr>
<td>2</td>
<td>2.CA.1 Add and subtract within 100</td>
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</tbody>
</table>
| 3           | 3.C.1 Add/subtract within 1000  
|             | 3.C.6 Multiplication facts 0-10  |
| 4           | 4.C.1 Add/subtract multi-digit numbers  
|             | 4.C.4 Multiply within 100  |
| 5           | 5.C.1 Multiply multi-digit whole numbers  |
| 6           | 6.C.1 Divide multi-digit whole numbers  |

**Strategies and Examples to Build Conceptual Understanding**

**Addition and Subtraction Examples**

- Use drawings and objects to physically manipulate numbers (two-color counters, tiles, blocks, whiteboards, etc.)
- Allow students to create flashcards with pictorial representations that accompany facts (note: flashcards alone are drill and practice methods. Having student add representations help build their conceptual understanding prior to practice.)
• 5, 10, and 20 frames –model sums in different ways to help students build understanding of how to decompose and compose in several ways.

![Image](image1.png)

- Develop an understanding of terminology such as: add, join, put together, combine, minus, take away, difference, compare
- Model problems as real-life word problems
- Vary word problem structures that match the scenarios
  - $3 + 2 = \square$ – Hannah had 3 cookies and her friend gave her 2 more. How many cookies does she have now?
  - $\square + 3 = 5$ – Sydney had some cookies and her mom gave her 3 more. She now had 5 cookies altogether. How many did Sydney start with?
  - $4 + \square = 5$ – William bought 4 pencils from the bookstore. When he got home, he found more pencils and now he had 5. How many did he have at home?
  - $5 - 1 = \square$ – Thomas had 5 kittens, but gave 1 away to his neighbor. How many does he have left?
  - $\square - 4 = 1$ – Mom gave my 3 friends and I a glass of lemonade. She had 1 glass left. How many glasses did she have to start with?
  - $5 - \square = 2$ – Dad had $5.00 and bought some candy at the gas station. He had $2.00 left. How much was the candy?
- Explicitly teach strategies like adding one, adding two, doubles and doubles plus one, doubles plus two, counting down, counting up, etc.
- Commutative property
- Vast experience with missing addends (not only giving addends to find the sum) - $\square + 4 = 7$; $3 + \square = 12$; etc.
- Thinking addition to support subtraction (thinking backwards)
- Fact families/turnaround facts taught simultaneously ($4 = 3 = 7$; $7 - 3 = 4$)
- Ten frames – model sums of 10 in different ways to help students build understanding of how to compose and decompose in several ways.
- Decompose numbers into friendly numbers (for 5+4, decompose 5 into 4 + 1 and recognize doubles (4 + 4 + 1 = 9)
- Encourage think addition by modeling “what goes with this part to make the total?”  For example, count out 13 counters and cover them up.  Remove 5 counters from that total, keeping those in view for students.  Think “five and what makes 13?”  Uncover the remaining counters to check.
- Use missing number cards – on index cards or number strips, make missing number trios. The circled number is always the sum.  How are the following related?  What number is missing?

\[
\begin{array}{c}
4 & \bigcirc & 8 \\
13 & \bigcirc & 7 \\
\end{array}
\]

- Arrays (example: 4 + 4 + 4 = 12)

\[
\begin{array}{c}
\cdot \\
\cdot \\
\cdot \\
\cdot \\
\cdot \\
\cdot \\
\cdot \\
\cdot \\
\end{array}
\]

- Base ten blocks (race to 100 or race to 0) – practices regrouping and carrying in addition and subtraction
- Number Lines

\[
\begin{array}{c}
37 + 48 \\
+ 40 + 30 + 30 + 25 \\
+ 30 + 30 + 30 + 25 \\
\end{array}
\]

- Bar Models

\[
\begin{array}{c}
\text{Girls} \\
\text{Boys} \\
? \\
23 \\
\end{array}
\]

151
**Multiplication and Division Examples**

- **Arrays** (4 x 4 = 16)

- **Fact Families** (3 x 5 = ?, 5 x 3 = ?, 15 ÷ 5 = ?, 15 ÷ 3 = ?)

- **Vary problem structures that match authentic scenarios**
  - 3 x 2 = □ – My sister and I both had 3 cookies. How many did we have together?
  - □ x 3 = 12 – I split my 12 cookies with my 2 friends and myself. How many cookies did each of us receive?
  - 4 x □ = 20 – My mom, dad, brother, and I went to the movies and it cost us $20.00 for tickets. How much did each ticket cost?
  - 14 ÷ 2 = □ – There were fourteen students in the class. Half of them went to the convocation in the gym and the other half stayed in the classroom for a movie. How many went to each event?
  - □ ÷ 4 = 6 – In our school, we have four tables in the cafeteria with six seats around each one. How many seats are there in all?
  - 25 ÷ □ = 5 – Dad had $25.00 to spend at the video rental store. He spent the entire amount on 5 movies. How much did each movie cost?

- **Equal groups** (4 x 3 = 12)
- Base ten blocks (model multiplying and dividing) – representing the problems \(24 \times 3\) and \(92 \div 3\)

- Number Lines (a way to represent \(4 \times 3 = 12\) and \(25 \div 5\))

- Bar Models
Decimals

- Estimate first
- Model with money
- Base Ten blocks for decimals

\[
0.3 \times 0.4 = 0.12
\]

\[
6.871 + 5.194
\]

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<thead>
<tr>
<th>Hundreds</th>
<th>Tens</th>
<th>Ones</th>
<th>Tenths</th>
<th>Hundredths</th>
<th>Thousandths</th>
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Conceptual Understanding Defined (two interpretations)

1. “Conceptual understanding refers to an integrated and functional grasp of mathematical ideas. Students with conceptual understanding know more than isolated facts and methods. They understand why a mathematical idea is important and the kinds of contexts in which it is useful. They have organized their knowledge into a coherent whole, which enables them to learn new ideas by connecting those ideas to what they already know. Conceptual understanding also supports retention. Because facts and methods learned with understanding are connected, they are easier to remember and use, and they can be reconstructed when forgotten. If students understand a method, they are unlikely to remember it incorrectly. They monitor what they remember and try to figure out whether it makes sense. They may attempt to explain the method to themselves and correct it if necessary. Although teachers often look for evidence of conceptual understanding in students’ ability to verbalize connections among concepts and representations, conceptual understanding need not be explicit. Students often understand before they can verbalize that understanding.”

http://www.nap.edu/openbook.php?isbn=0309069955&page=118

2. “Students demonstrate conceptual understanding in mathematics when they provide evidence that they can recognize, label, and generate examples of concepts; use and interrelate models, diagrams, manipulatives, and varied representations of concepts; identify and apply principles; know and apply facts and definitions; compare, contrast, and integrate related concepts and principles; recognize, interpret, and apply the signs, symbols, and terms used to represent concepts. Conceptual understanding reflects a student's ability to reason in settings involving the careful application of concept definitions, relations, or representations of either.”


www.illustrativemathematics.org

www.insidemathematics.org

http://www.doe.in.gov/achievement/curriculum/mathematics-toolboxes (opens a zip file for each grade that has further examples and explanations of standards)

http://www.thedailyriff.com/WordProblems.pdf - explanations and examples of the Singapore Bar Model

Alternative Algorithms - http://everydaymath.uchicago.edu/teaching-topics/computation/
**Suggested books and reading**

*Teach Student-Centered Mathematics* *(available in multiple grade bands)* by John A. Van de Walle & LouAnn Lovin


http://www.amazon.com/Teaching-Student-Centered-Mathematics-Grades-Volume/dp/0205408443/ref=pd_bxgy_b_text_y

*Number Talks* by Sherry Parrish


https://wiki.eee.uci.edu/index.php/Multiplication_&_Division__Building_Conceptual_Understanding-Section_A


